Find the exact form of solution for $\sqrt{3x-4} + \sqrt{2x+5} = 5$

Solution: step 1. Separate radicals:

$\sqrt{3x-4} = 5 - \sqrt{2x+5}$

Step 2. Square both sides: $(\sqrt{3x-4})^2 = (5 - \sqrt{2x+5})^2$

Step 3. Simplify:

$3x - 4 = 25 - 10\sqrt{2x+5} + 2x + 5$

$3x - 4 = 30 + 2x - 10\sqrt{2x+5}$

$3x - 2x - 4 - 30 = -10\sqrt{2x+5}$

$x - 34 = -10\sqrt{2x+5}$

Step 4. Square both sides again:

$(x - 34)^2 = (-10\sqrt{2x+5})^2$

$x^2 - 68x + 1156 = 100(2x + 5)$

$x^2 - 268x + 656 = 0$

Step 5. Solve quadratic equation using formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 1$, $b = -268$, $c = 656$

$x = \frac{-(-268) \pm \sqrt{(-268)^2 - 4(1)(656)}}{2(1)} = \frac{268 \pm \sqrt{69200}}{2}$

$\sqrt{69200} = \sqrt{(4^2)(5^2)(173)} = 20\sqrt{173}$, $x = \frac{268 \pm 20\sqrt{173}}{2}$

$x = 134 \pm 10\sqrt{173}$ \( \Rightarrow \) The exact form of solution.

Possible
Step 6. Check: Substitute $x_1 = 134 + 10\sqrt{173}$ to equation:

R.H.S = 5, L.H.S = $\sqrt{3(134+10\sqrt{173}) - 4} + \sqrt{2(134+10\sqrt{173}) + 5} \neq 5$

$\therefore x_1 = 134 + 10\sqrt{173}$ is not the solution of the equation.

Step 7. Check: Substitute $x_2 = 134 - 10\sqrt{173}$ to equation:

R.H.S = 5, L.H.S = $\sqrt{3(134-10\sqrt{173}) - 4} + \sqrt{2(134-10\sqrt{173}) + 5} = 5$

$\therefore x_2 = 134 - 10\sqrt{173}$ is the solution in exact form.

The approximate solution is $x_2 = 2.047$