The general linear differential equation of the $n^{th}$ order  

$$a_0y^{(n)} + a_1y^{(n-1)} + \cdots + a_{n-1}y^{(1)} + a_ny = b$$

If $b = 0$, the general linear differential equation is called Homogeneous.

If $b \neq 0$, the general linear differential equation is called nonhomogeneous.

The second – order homogeneous equation with constant coefficients ($a_0, a_1, \text{and } a_2$) is of the form

$$a_0y'' + a_1y' + a_2y = 0$$

To get the general solutions, follow the following steps:

Step1. Write the Auxiliary Equation: $a_0m^2 - a_1m - a_2 = 0$

Step2. Find the solution of $a_0m^2 - a_1m - a_2 = 0$, $m_1, m_2$

Step3. The general solution for the differential equation will be as follows:

$$y = c_1e^{m_1x} + c_2e^{m_2x}, \text{ if } m_1, m_2 \text{ are two different real numbers.}$$

$$y = e^{mx}(c_1 + c_2x), \text{ if } m_1 = m_2, \text{ and also real number.}$$

$$y = c_1e^{ax}(c_1e^{jbx} + c_2e^{-jbx}), \text{ if the Auxiliary Equation has two complex number roots:}$$

$$m_1 = a + jb, \ m_1 = a - jb, a \text{ is the real number portion}$$

Or, the general solution also can be expressed as trigonometry form:

$$y = e^{ax}(c_1\cos bx + c_2\sin bx)$$

To get the Particular solutions, follow the following steps:

Step1. Apply the given inertial information to the general solution to solve constant $c_1$ and $c_2$.

Step2. Substitute $c_1$ and $c_2$ back to the general solution.

For details, please read question 5 in solution.