Calculus Worksheet

Sketching a function using Calculus

Practice Questions

Sketching function: \( y = 2x^3 + 3x^2 - 36x - 10 \) using Calculus

Step 1, find the y intercept:

Let \( x = 0 \), so \( y_{\text{intercept}} = -10 \)

Step 2, find the x intercepts:

Let \( y = 0 \), so solve equation: \( 2x^3 + 3x^2 - 36x - 10 = 0 \),

\[ \therefore x_{\text{intercept}1} = -0.27, \quad x_{\text{intercept}2} = -4.9, \quad x_{\text{intercept}3} = 3.7 \]

Step 3, find the first derivative for \( y = 2x^3 + 3x^2 - 36x - 10 \)

\[ \frac{dy}{dx} = 6x^2 + 6x - 36 \]

Let \( \frac{dy}{dx} = 0 \), solve \( 6x^2 + 6x - 36 = 0 \),

\[ 6(x^2 + x - 6) = 0, \]

\[ (x - 2)(x + 3) = 0, \]

\[ \therefore x_1 = 2, \quad x_2 = -3 \quad \text{These are the critical points.} \]

Step 4, find the second derivative to identify the MAX or MIN point:

Plug \( x_1 = 2 \) into the second derivative:

\[ \frac{d^2y}{dx^2} = 12(2) + 6 = 30 > 0, \quad \therefore x_1 = 2 \text{ is the MIN point} \]

\[ y(2) = 2(2)^3 + 3(2)^2 - 36(2) - 10 = -54 \]

\[ \therefore \text{the MIN point is} \ x = 2, \ y = -54 \]
Plug $x_2 = -3$ into the second derivative:

$$\frac{d^2y}{dx^2} = 12(-3) + 6 = -36 + 6 = -30 < 0, \quad \therefore x_2 = -3 \text{ is the MAX point}$$

its y-coordinate: $y(-3) = 2(-3)^3 + 3(-3)^2 - 36(-3) - 10 = 71$

$\therefore \text{ the MAX point is } x = -3, \quad y = 71$

Step 5, find the inflection point by letting $\frac{d^2y}{dx^2} = 0$,

$$\frac{d^2y}{dx^2} = 12x + 6 = 0,$$

$$6(2x + 1) = 0, \quad \therefore x = -0.5 \text{ is the inflection point}$$

its y-coordinate: $y(0.5) = 2(-0.5)^3 + 3(-0.5)^2 - 36(-0.5) - 10 = 8.5$

Note: The inflection point is always between the MAX and MIN point.