Example 1:

Find the equation of the line tangent to \( y = 4 - x^2 \) at \( x = 1 \)

NOTE: When we find the first derivative of an equation, we have solved the slope at every point.

Steps to finding the equation of a tangent or normal line:

1. Find the first derivative of the function. The first derivative is the slope at every point.
2. Plug \( x \) into the first derivative to find the slope at that particular \( x \) value.
3. Solve for \( y \) using the original function.
4. Use \( m = \frac{y - y_1}{x - x_1} \) to solve for the equation.

Step by step instruction:

1. Find the derivative of \( y = 4 - x^2 \). \( \Rightarrow \frac{dy}{dx} = -2x \).
   
   \(-2x\) is the slope of the function at every point.

   Find the slope by plugging \( x = 1 \) into the derivative.

   \[
   \frac{dy}{dx}|_{x=1} = -2(1) = -2 = \text{slope (m)}
   \]

2. Next, using \( x = 1 \), solve for \( y \). \( y = 4 - x^2 \) \( \Rightarrow \) \( y = 4 - (1) = 3 = y \)

   Using \( m = \frac{y - y_1}{x - x_1} \), plug in \( m, x, \) and \( y \) to solve the equation.

   \[
   m = \frac{y - y_1}{x - x_1} = \Rightarrow m = \frac{y - y_1}{x - x_1} \Rightarrow -2 = \frac{y - 3}{x - 1}
   \]

   \(-2(x - 1) = y - 3 \) \( \Rightarrow \) \(-2x + 2 = y - 3 \) \( \Rightarrow \) \( y = -2x + 5 \) (in slope-intercept form)

   Therefore, \( y = -2x + 5 \) is the equation of the tangent line at \( x = 1 \).
Find the equations for the tangent lines at the point indicated (in general form).

1. \( y = 3x^2 - 2x + 1 \) at \( x = 1 \)

2. \( y = x^3 - 7x + 4 \) at \( x = -1 \)

3. \( x^2 + y^2 = 36 \) at \( (3, 3\sqrt{3}) \)

4. \( y = \frac{3}{(x^2 - 1)^2} \) at \( \left(2, \frac{1}{3}\right) \)

5. \( y^2 = 50x - 1 \) for \( (1, 7) \)

6. \( 3x - x^2 + 2y = 7 \) at \( x = -2 \)

ANSWERS:

1) \( 4x - y - 2 = 0 \)
2) \( 4x + y + 14 = 0 \)
3) \( x + \sqrt{3}y - 12 = 0 \)
4) \( 8x + 9y - 19 = 0 \)
5) \( 25x - 7y + 24 = 0 \)
6) \( 7x + 2y - 3 = 0 \)