Solution: Exercises 26.6

5. The spring of a spring balance is 8.0 cm long when there is 0 kg weight on the balance, and it is 9.5 cm long with 6.0 N hung from the balance. How much work is done in stretching it from 8.0 cm to a length of 10.0 cm?

Step 1: Find the spring constant:

\[ F = -Kx \]

\[ 6.0 \text{ N} = K (1.5 \text{ cm}) \]

\[ \Rightarrow K = 4.0 \text{ N/cm} \]

Step 2: \[ W = F \cdot d \]

\[ W = Kx \cdot dx \]

\[ W = \int Kx \, dx \]

\[ x \] is the stretching \[ W = \int_0^3 4.0 \cdot x \, dx \]

\[ W = 4.0 \left[ \frac{x^2}{2} \right]_0^3 = 2 \left[ (3^2 - 0) \right] = 8.0 \text{ N} \cdot \text{m} \]
Solution: Exercise 26.6, #18.

A rectangular swimming pool full of water is 5.50 m wide, 13.5 m long, and 1.75 m deep. Find the work done in pumping the water from the pool to a level 1.25 m above the top of the pool.

\[\text{work } W = F \cdot d,\]

here \(F\) is the weight of water.

To pumping water from the pool to 1.25 m above the top of pool,

\[dV = (\text{Length})(\text{Width})(\text{small thickness of depth})\]

\[\therefore dV = (13.5 \text{ m})(5.50 \text{ m})(dy)\]

\[d(\text{weight}) = dV \cdot \gamma \quad (\gamma = 9.8 \text{ KN/m}^3)\]

\[F = 728.39 \text{ dy}\]

\(d\) is the distance of a piece of water at certain position to be pumped up to 3.00 m. \((1.75 \text{ m} + 1.25 \text{ m} = 3.00 \text{ m})\)

\[W = \int_0^{3.00} (728.39 \text{ dy})(3-y)\]

\[W = 728.39 \int_0^{3.00} (3-y)dy = 728.39 \left[3y - \frac{y^2}{2}\right]_0^{3.00}\]

\[W = 728.39 \left[ (3(3.00) - \frac{3(0)^2}{2}) - (0) \right] = 3.28 \times 10^3 \text{ (KN.m)}\]
Solution: Exercises 26.6

# 35. The length of arc $s$ of a curve from $x = a$ to $x = b$ is

\[ s = \int_{a}^{b} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \]

The cable of a bridge can be described by the equation

\[ y = 0.04 \ x^{3/2} \] from $x = 0$ to $100 \ m$. Find the length of the cable.

\[ \begin{align*}
\frac{dy}{dx} &= 0.04 \left( \frac{3}{2} \right) x^{1/2} \\
\left( \frac{dy}{dx} \right)^2 &= (0.06 x^{1/2})^2 = 0.036 x
\end{align*} \]

\[ \begin{align*}
s &= \int_{0}^{100} \sqrt{1 + 0.036 x} \, dx \\
&= \int_{1}^{100} \sqrt{1 + 0.036 u} \, du \\
&= \frac{10^3}{1.8} \int_{1}^{100} \frac{1}{u} \, du \\
&= \frac{10^3}{1.8} \left[ \ln(u) \right]_{1}^{100} \\
&= \frac{10^3}{1.8} \left[ \ln(100) - \ln(1) \right] \\
&= \frac{10^3}{1.8} \ln(100) \\
&= \frac{10^3}{1.8} \cdot 4.605 \\
&= 108.5 \ m
\end{align*} \]

The cable is 108.5 m long.