Find the Derivative of a Function by Different Methods (2)

Find \( \frac{dy}{dx} \) for \( y = \frac{2x}{(x-1)^2} \) by Implicit Differential

Solution: Step 1, Rewrite the given function into \( y(x-1)^2 = 2x \) as an implicit function.

Step 2, Derivative for both side:

\[
\frac{d}{dx} [y(x-1)^2] = \frac{d}{dx} (2x)
\]

\[
(1) \frac{dy}{dx}(x-1)^2 + 2y(x-1)(1) = 2
\]

\[
\frac{dy}{dx} (x-1)^2 = 2 - 2y(x-1)
\]

\[
\frac{dy}{dx} = \frac{2-2y(x-1)}{(x-1)^2} \quad \text{Answer}
\]

We can get the exact same answer with the other methods if we substitute \( y = \frac{2x}{(x-1)^2} \) to this answer:

\[
\frac{dy}{dx} = \frac{2-2(x-1)\frac{2x}{(x-1)^2}}{(x-1)^2}
\]

\[
= \frac{2-4x}{(x-1)^2} = \frac{2-4x}{(x-1)^2} \cdot \frac{(x-1)}{(x-1)} = \frac{2(x-1)-4x}{(x-1)^3} = \frac{-2x-2}{(x-1)^3} = \frac{2(1+x)}{(1-x)^3}
\]

\[
\frac{dy}{dx} = \frac{2(1+x)}{(1-x)^3}
\]