Divisibility Tests

• A shortcut to determine a given number if it is divisible by another number.
• An important skill in reducing fractions, finding factors especially for larger numbers.
• A shortcut for predicting the remainder in solving long division questions.
Divisible by 1

• Any integer is divisible by 1.
Divisible by 2

• If it is an even number (the last digit is even)

Example:

30, 972, 1954 are all even numbers. They are divisible by 2.

For reducing fractions:

E.g.1, \[
\frac{8}{12} = \frac{4}{6} = \frac{2}{3}
\]

E.g.2, \[
\frac{16}{80} = \frac{8}{40} = \frac{4}{20} = \frac{2}{10} = \frac{1}{5}
\]
Divisible by 3

- If the sum of all digits is divisible by 3.

Example: To test 564: 5 + 6 + 4 = 15, 15 \( \div 3 = 5 \), so 564 is divisible by 3.

To test 1701: 1 + 7 + 0 + 1 = 9, 9 \( \div 3 = 3 \), so 1701 is divisible by 3.

For reducing fractions:

E.g. 1, \( \frac{9}{39} = \frac{3}{13} \)  
E.g. 2, \( \frac{15}{18} = \frac{5}{6} \)  
E.g. 3, \( \frac{33}{57} = \frac{11}{19} \)
Divisible by 4

- If the last two digits is divisible by 4.

Example:

To test 9024 : 24 4=12, so 9024 is divisible by 4.

To test 67116 : 16 4=4, so 67116 is divisible by 4.

To test 109380: 80 4= 20, so 109380 is divisible by 4.

For reducing fractions:

E.g.1, \[ \frac{20}{144} = \frac{5}{36} \]  E.g.2, \[ \frac{44}{148} = \frac{11}{37} \]
Divisible by 5

• If the last digit is 0 or 5.

Example:

260, 2970 are divisible by 5. (last digit is zero)

75, 57835 are divisible by 5. (last digit is 5)

For reducing fractions:

E.g.1, \( \frac{10}{15} = \frac{2}{3} \)  

E.g.2, \( \frac{165}{250} = \frac{35}{50} = \frac{7}{10} \)
Divisible by 6

• If the number is divisible by 2 and 3.

Example,

456 is an even number → it is divisible by 2.
4+5+6 = 15, 15 3 =5 → it is divisible by 3,
so 456 is divisible by 6.

3198 is an even number → it is divisible by 2.
3+1+9+8 = 21, 21 3=7 → it is divisible by 3,
so 3198 is divisible by 6.
Divisible by 6

For reducing fractions:

E.g. 1, \[ \frac{36}{48} = \frac{6}{8} = \frac{3}{4} \]

36 is even number, and 3+6=9 is divisible by 3, so 36 is divisible by 6.

48 is even number, and 4+8=12 is divisible by 3, so 48 is divisible by 6.

*When the number gets bigger, just divided by 2 first, and then divided by 3.

E.g. 2, \[ \frac{138}{246} = \frac{138 \div 2}{246 \div 2} = \frac{69}{123} = \frac{23}{41} \]
Divisible by 7

• Method 1, (when a number has less than six digits.) Double the last digit and subtract it from the rest of the number to see if the result is divisible by 7. The process can be repeated as many time as needed.

Example, To test 196: 19 – (2 6) = 7, 7 7 = 1, so 196 is divisible by 7.

To test 16205: 1620 – (2 5) = 1610,
again, 161 – (2 0) = 161,
again, 16 – (2 1) = 14, 14 2 = 7,
so 16205 is divisible by 7.
Divisible by 7

• Method 2, (when a number has more than 6 digits).

Example, to test 321461: 321461-321321=140, 140 \( \div 7 = 20 \), so 321461 is divisible by 7.

Why do I subtract 321321 from 321461?

1) Any six digits number with the pattern as \( xyzxyz \) is divisible by 7. Hence, 321321 is multiple of 7.

2) 321321 is the closest and largest multiple of 7 to 321461.
Divisible by 8

• If the number formed by the last three digits of a given number is Divisible by 8.

• Example, To test 5,160: 160 \( \div 8 = 20 \), so 5,160 is divisible by 8.

To test 17,248: 248 \( \div 8 = 31 \), so 17,248 is divisible by 8.

* The alternate way is keep dividing by 2 if the result is still an even number.
Divisible by 9

• If the sum of all the digits of a given number is divisible by 9.

Example,

To test 927: 9+2+7 = 18, 18/9 = 2, so 927 is divisible by 9.

To test 87,255: 8+7+2+5+5 = 27, 27/9 = 3,

So 87,255 is divisible by 9.
Divisible by 10

• If the last digit is 0.
• Example,
  40, 120, 3570, 39860, 700000 are all divisible by 10.
Divisible by 11

• If the difference between the sum of odd digits and the sum of the even digits is zero, or divisible by 11.

Example,

To test 1595: the sum of odd digits = 1+9=10
the sum of even digits = 5+5=10
the difference = 10-10=0, so 1595 is divisible by 11.

To test 23871903: the sum of odd = 2+8+1+0=11,
the sum of even = 3+7+9+3=22,
the difference = 22-11=11,
So, 23871903 is divisible by 11.
Summary

• Numbers are divisible by 2, 4, 6, and 8, they must be even numbers (the last digit are 2, 4, 6, 8, or 0). In other word, if the last digit is an odd, then it must not divisible by any of 2, 4, 6, or 8.

• If a number is divisible by 9, then it must divisible by 3. When reducing fractions, breaking into two steps are acceptable.

• If an even number is divisible by 3, then it must divisible by 6. When reducing fractions, breaking into two steps are acceptable.

• The Divisibility Test for 7 is unique, as well as divisibility test for 11. They follow different patterns.